Team Round

GLMT 2025

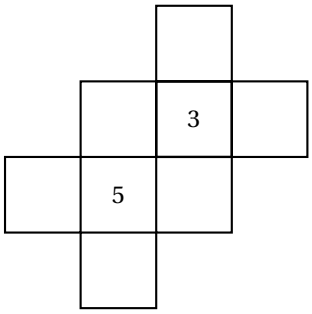
April 19, 2025

1. **[20]** In a right triangle, the smallest angle is π degrees. Find the largest angle in the triangle, in degrees.

2. **[25]** A 12 by 20 rectangle is split into four congruent rectangles. Find the largest possible perimeter of one of the four rectangles.

3. **[25]** There exists one non-degenerate triangle with integer side lengths such that the product of its sides equals 210. Find the perimeter of this triangle.

4. **[30]** In the figure below, if a cell has more than one neighbor, its value is equal to sum of its neighbors. Find the sum of the values in the 6 empty cells.



5. **[30]** Suppose n is a positive integer such that if a fair n-sided die is rolled, the probability it rolls a square number is $\frac{1}{7}$. Find the sum of all possible values of n.

6. **[35]** Sylvia marks a point A in the Cartesian Plane. Lena starts at the lattice point closest to A, then travels in straight lines to the second closest, the third closest, and finally the fourth closest lattice points to A. Find the minimum possible length of the path Lena travels. Assume that no two of the four closest lattice points are equidistant to the point Sylvia selects.

7. **[35]** There are exactly two ordered pairs of real numbers (x, y) that satisfy $y = x^2 - 7x + 15$ and $x = y^2 - 7y + 15$. Find the sum of the values of x for these two ordered pairs.

8. **[40]** Find the sum of the digits of

$$9 \cdot (1 + 22 + 333 + 4444 + 55555 + 666666 + 7777777 + 88888888 + 999999999).$$

9. **[45]** Aurora, Bertha, Candice, and Denise each think of a distinct integer from 1 to 20, inclusive. They do not know what numbers the others are thinking of. They have the following perfectly logical conversation.

   - Aurora: It is possible that my number divides all of your numbers.
   - Bertha: My number could've as well, but now it definitely can't.
   - Candice: My number is definitely a multiple of someone else's number. However, one of you may be thinking of a number bigger than mine.
   - Denise: You are right, your number is a multiple of both Aurora and Bertha's numbers.
   - Aurora: I now know everyone's numbers!

Find the product of their four numbers.

10. **[45]** Each term of the sequence

$$2, 3, 10, \ldots$$

is a product of the corresponding terms from two fixed arithmetic sequences. Find the 10th term of this sequence.

11. **[50]** Emmy puts the numbers 1 through 25 in the cells of a 5×5 grid. A cell is called *extreme* if it contains the smallest or largest number in its row or column. Find the maximum possible number of extreme cells.

12. **[50]** Let x, y, and z be positive real numbers satisfying

$$\begin{aligned}(x+y)(x+z) &= 20,\\ (x+z)(z-x) &= 4, \text{ and}\\ (y-z)(y+z) &= 6.\end{aligned}$$

Find $x+y+z$.

13. **[55]** Find the value of

$$\prod_{k=0}^{4}\left(2^{2^k}+\frac{1}{2^{2^k}}-1\right).$$

14. **[55]** In triangle ABC, let D, E, and F be the points where the incircle is tangent to sides BC, AC, and AB, respectively. Given that $AE = 7$, $BF = 8$, and $CD = 9$, find $\cos(\angle FDE) \cdot \cos(\angle DEF) \cdot \cos(\angle EFD)$.

15. **[60]** In a 5×5 grid, Sasha draws paths between 4 distinct adjacent cells (not including diagonally adjacent), where the order of the cells matters. Find the number of paths Sasha draws.